$$\begin{vmatrix} x & y & -z \\ y & -z & x \\ -z & x & y \end{vmatrix}^3 = \begin{vmatrix} X & Y & Z \\ Y & Z & X \\ Z & X & Y \end{vmatrix} \quad \ldots.(2),$$

where
$$X=x^3+y^3-z^3-3xyz$$
$$Y=3(x^2y+xz^2-y^2z)=0.$$
$$Z=3(xy^2+yz^2-x^2z)$$

Then substituting in (2) and expanding

$$X^3+Z^3=(x^3+y^3-z^3+3xyz)^3$$

which is impossible, since the sum of two integral cubes cannot be an integral cube. (For a proof, see Euler's *Algebra.*) Hence the impossibility of (1) is established.

Also solved by the late *JOSIAH H. DRUMMOND.*

AVERAGE AND PROBABILITY.

122. Proposed by F. M. PRIEST, St. Louis, Mo.

Suppose each of the nine digits to be placed in a wheel, and five of them drawn at random therefrom, and written down in the order drawn. What is the probability the number thus expressed will be greater than 50,000?

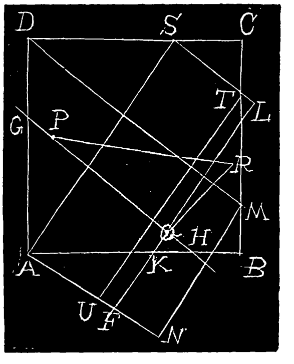
Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and J. F. LAWRENCE, A. B., Breckenridge, Mo.

If 5, 6, 7, 8, or 9 be drawn first, the number will be greater than 50,000.

The chance of drawing 5 is $\frac{1}{9}$; of drawing 6, $\frac{1}{9}$; of drawing 7, $\frac{1}{9}$; of drawing 8, $\frac{1}{9}$; of drawing 9, $\frac{1}{9}$. The chance of drawing 5, 6, 7, 8, or 9 is, therefore, $\frac{5}{9}$. Therefore the chance that the number is greater than 50,000 is $\frac{5}{9}$.

123. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Three points are taken at random within a square. What is the probability that the angle formed by joining them is acute?



Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let $ABCD$ be the square; P, Q, R the three random points. Through P, Q draw GH, and through Q draw KL perpendicular to GH. When P is between G and Q the angle PQR will be obtuse if R lies on the opposite side of LK from P. Draw AF perpendicular to KL; AES perpendicular to GH; ST parallel to GH; MN and TU parallel to KL; DM parallel to GH.

Let $AB=a$, $PQ=x$, $AF=y$, $AE=z$, $\angle KAF=\theta$. Then $AK=y\sec\theta$, $BK=$

$a-y\sec\theta$, when L is on BC, $BL=(a-y\sec\theta)\cot\theta$, when L is on CD, and $CL=a(1-\tan\theta)-y\sec\theta$. Area $LBK=\frac{1}{2}(a-y\sec\theta)^2\cot\theta=\frac{1}{2}u$. Area $LCBK=\frac{1}{2}a(2a-2y\sec\theta-a\tan\theta)=\frac{1}{2}u_1$. An element of area at P is $xdxd\theta$; at Q, $dydz$. The limits of θ for $\frac{1}{2}u$ are $\frac{1}{4}\pi$ and $\frac{1}{2}\pi$, and 0 and $\frac{1}{4}\pi$; for $\frac{1}{2}u_1$, 0 and $\frac{1}{4}\pi$; of y for $\theta=\frac{1}{4}\pi$ to $\theta=\frac{1}{2}\pi$, $y=0$ to $y=a\cos\theta$; for $\theta=0$ to $\theta=\frac{1}{4}\pi$, $y=AN=a(\sec\theta-\sin\theta)=y_2$ to $y=a\cos\theta$, and $y=AU=a(1-\tan\theta)\sec\theta=y_3$ to $y=y_2$, and $y=a(\cos\theta-\sin\theta)=y_1$ to $y=y_3$, for $\frac{1}{2}u_1$, $y=0$ to $y=y_1$; of z, for $y=0$ to $a\cos\theta$, $z=y\tan\theta$ to $z=a\cos\theta$ and $z=a\cos\theta$ to $z=a\operatorname{cosec}\theta-y\cot\theta=z_1$; for $y=y_2$ to $y=a\cos\theta$, $z=y\tan\theta$ to $z=z_1$, for $y=y_3$ to $y=y_2$, $z=y\tan\theta$ to $z=a\cos\theta$ and $z=a\cos\theta$ to $z=z_1$, for $y=y_1$ to $y=y_3$, $z=y\tan\theta$ to $z=a\cos\theta$ and $z=a\cos\theta$ to $z=a\sec\theta$, for $\frac{1}{2}u_1$ the limits of z are $y\tan\theta$ to $a\cos\theta$ and $a\cos\theta$ to $a\sec\theta$; of x, 0 and $y+z\tan\theta=x_1$, and 0 and $y+a\operatorname{cosec}\theta-z\cot\theta=x_2$. The limits of x must be doubled for the case when P is on the opposite side of LK. The whole number of ways the three points can be taken is a^6. Let p' be the chance that PQR is obtuse.

$$p'=\frac{1}{a^6}\left[\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}\int_0^{a\cos\theta}ud\theta du+\int_0^{\frac{1}{4}\pi}\int_{y_3}^{y_2}ud\theta du\right]\left[\int_{y\tan\theta}^{a\cos\theta}\int_0^{x_1}dzxdx+\int_{a\cos\theta}^{z_1}\int_0^{x_2}dzxdx\right]$$

$$+\frac{1}{a^6}\left[\int_0^{\frac{1}{4}\pi}\int_{y_1}^{y_3}ud\theta du+\int_0^{\frac{1}{4}\pi}\int_0^{y_1}u_1d\theta du\right]\left[\int_{y\tan\theta}^{a\cos\theta}\int_0^{x_1}dzxdx+\int_{a\cos\theta}^{a\sec\theta}\int_0^{x_2}dzxdx\right]$$

$$+\frac{1}{a^6}\int_0^{\frac{1}{4}\pi}\int_{y_2}^{a\cos\theta}\int_{y\tan\theta}^{z_1}\int_0^{x_1}ud\theta dydzxdx=\frac{1}{6a^6}\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}\int_0^{a\cos\theta}[(y+a\sin\theta)^3\sec\theta\operatorname{cosec}\theta-$$

$$(y+a\sin\theta-a\cos\theta)^3\operatorname{cosec}^5\theta\sec\theta-y^3\sec^5\theta\operatorname{cosec}\theta]ud\theta dy$$

$$+\frac{1}{6a^6}\int_0^{\frac{1}{4}\pi}\left[\int_{y_3}^{y_2}[(y+a\sin\theta)^3\sec\theta\operatorname{cosec}\theta-y^3\sec^5\theta\operatorname{cosec}\theta-(y+a\sin\theta-a\cos\theta)^3\right.$$

$$\operatorname{cosec}^5\theta\sec\theta]udy$$

$$+\int_{y_1}^{y_3}[(y+a\sin\theta)^3\sec\theta\operatorname{cosec}\theta-y^3\sec^5\theta\operatorname{cosec}\theta-y^3\sin\theta\sec\theta]udy$$

$$+\int_0^{y_1}[(y+a\sin\theta)^3\sec\theta\operatorname{cosec}\theta-y^3\sec^5\theta\operatorname{cosec}\theta-y^3\sin\theta\sec\theta]u_1dy$$

$$\left.+\int_{y_2}^{a\cos\theta}(a^3\sec^2\theta\operatorname{cosec}\theta-y^3\sec^5\theta\operatorname{cosec}\theta)udy\right]d\theta$$

$$=\frac{1}{360}\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}(14\sin\theta\cos\theta+14\cos^2\theta+6\cot\theta+\cot^2\theta-\operatorname{cosec}^2\theta-20\cot\theta\operatorname{cosec}^2\theta$$

$$+45\cot^2\theta\operatorname{cosec}^2\theta-36\cot^3\theta\operatorname{cosec}^2\theta+10\cot^4\theta\operatorname{cosec}^2\theta)d\theta$$

$$+\tfrac{1}{360}\int_0^{\frac{1}{4}\pi}(4\sin^2\theta+2\cos^2\theta+42\sin\theta\cos\theta-12\sin\theta\cos^3\theta-20\sin^3\theta\cos\theta+\sec^8\text{cosec}^2\theta$$

$$+6\sec^7\theta\,\text{cosec}\,\theta-60\sec^2\theta\,\text{cosec}\,\theta-6\sec\theta\,\text{cosec}\,\theta-36\tan\theta+10\tan^2\theta$$

$$+20\tan^3\theta-\cot^2\theta+20\,\text{cosec}^2\theta+123\sec^2\theta-50\tan\theta\sec^2\theta+5\tan^2\theta\sec^2\theta$$

$$+14\tan^3\theta\sec^2\theta-103\tan^4\theta\sec^2\theta+186\tan^5\theta\sec^2\theta-236\tan^6\theta\sec^2\theta$$

$$+192\tan^7\theta\sec^2\theta-60\tan^8\theta\sec^2\theta+36\tan^9\theta\sec^2\theta+84\tan^{10}\theta\sec^2\theta$$

$$+45\tan^{12}\theta\sec^2\theta)d\theta=\tfrac{119927}{400400}-\tfrac{11}{144}\log 2.$$

$$3p'=\tfrac{359781}{400400}-\tfrac{11}{48}\log 2,\ p=1-3p'=\tfrac{11}{48}\log 2+\tfrac{40619}{400400}.\quad\therefore\ p=.260292.$$

124. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Find the average area of a spherical polygon of $n=6$ sides.

No solution of this problem has been received.

MISCELLANEOUS.

118. Proposed by O. W. ANTHONY, New York, N. Y.

If f is determined by the equation $f(\mu\nu)=f(\mu)f^{-1}(\nu)+f(\nu)f^{-1}(\mu)$, where f^{-1} is the inverse of f, show that $f[(2)^\mu]=\dfrac{k^{\mu+1}}{2^{\mu+1}}$, where k is a constant.

Solution by G. B. M, ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$f(\mu\nu)=ff^{-1}(\mu\nu)+ff^{-1}(\mu\nu)$, but $ff^{-1}=1$. $\therefore f(\mu\nu)=2(\mu\nu)$. $\therefore f=2$.

$\therefore (2)^\mu=(f)^\mu$ or $f[(2)^\mu]=(f)^{\mu+1}=(2)^{\mu+1}$.

$\therefore f[(2)^\mu]=(\frac{1}{2}k)^{\mu+1}$, where $k=4$.

119. Proposed bv L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Show how to determine the illumination at any point of the surface of the water at the bottom of a deep well, due to the light from the sky.

A solution of this problem appeared in the November number. The problem was incorrectly numbered. ED.

120. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Prove $\Sigma\cos^4x-2\Pi\cos^2x+2\Pi\sin^2x=1-\sin(\Sigma)\sin\Pi(y+z-x)$.

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$$\sin(\Sigma)\sin\Pi(y+z-x)$$

$$=\sin(x+y+z)\sin(x+y-z)\sin(z-y+x)\sin(z+y-x)$$